

# Package ‘DiscreteWeibull’

January 20, 2025

**Type** Package

**Title** Discrete Weibull Distributions (Type 1 and 3)

**Version** 1.1

**Date** 2015-10-15

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**Description** Probability mass function, distribution function, quantile function, random generation and parameter estimation for the type I and III discrete Weibull distributions.

**License** GPL

**LazyLoad** yes

**Depends** Rsolnp, stats

**NeedsCompilation** no

**Repository** CRAN

**Date/Publication** 2015-10-18 01:21:16

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DiscreteWeibull-package

*Discrete Weibull Distributions (Type 1 and 3)*

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**Description**

Probability mass function, distribution function, quantile function, random generation and parameter estimation for the type I and III discrete Weibull distributions. In the present version, some modifications have been made on the functions in [Edweibull](#) for the computation of the moments of the type I discrete Weibull distribution.

**Details**

Package:	DiscreteWeibull
Type:	Package
Version:	1.1
Date:	2015-10-17
License:	GPL
LazyLoad:	yes
Depends:	Rsolnp

**Author(s)**

Alessandro Barbiero <alessandro.barbiero@unimi.it>

**References**

- T. Nakagawa and S. Osaki (1975) The discrete Weibull distribution, *IEEE Transactions on Reliability*, 24(5), pp. 300-301
- D.N.P. Murthy, M. Xie, R Jiang (2004) *Weibull models*, John Wiley & Sons: Hoboken, New Jersey
- W.J. Padgett and J.D. Spurrier (1985) Discrete failure models, *IEEE Transactions on Reliability*, 34(3), pp. 253-256
- H. Rinne (2008) *The Weibull Distribution: A Handbook*. CRC Press: Boca Raton, Florida
- M. S. A. Khan, A. Khaliq, and A. M. Abouammoh (1989) On estimating parameters in a discrete Weibull distribution, *IEEE Transactions on Reliability*, 38(3), pp. 348-350

**See Also**

[ddweibull](#), [estdweibull](#), [Edweibull](#), [ddweibull13](#), [estdweibull13](#), [Edweibull13](#)

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 Discrete Weibull (Type 1)

*The type 1 discrete Weibull distribution*


---

### Description

Probability mass function, distribution function, quantile function and random generation for the discrete Weibull distribution with parameters  $q$  and  $\beta$

### Usage

```
ddweibull(x, q, beta, zero = FALSE)
pdweibull(x, q, beta, zero = FALSE)
qdweibull(p, q, beta, zero = FALSE)
rdweibull(n, q, beta, zero = FALSE)
```

### Arguments

x	vector of quantiles
p	vector of probabilities
q	first parameter
beta	second parameter
zero	TRUE, if the support contains 0; FALSE otherwise
n	sample size

### Details

The discrete Weibull distribution has probability mass function given by  $P(X = x; q, \beta) = q^{(x-1)\beta} - q^{x\beta}$ ,  $x = 1, 2, 3, \dots$ , if zero=FALSE; or  $P(X = x; q, \beta) = q^{x\beta} - q^{(x+1)\beta}$ ,  $x = 0, 1, 2, \dots$ , if zero=TRUE. The cumulative distribution function is  $F(x; q, \beta) = 1 - q^{x\beta}$  if zero=FALSE;  $F(x; q, \beta) = 1 - q^{(x+1)\beta}$  otherwise

### Value

ddweibull gives the probability function, pdweibull gives the distribution function, qdweibull gives the quantile function, and rdweibull generates random values.

### Author(s)

Alessandro Barbiero

**Examples**

```

# Ex.1
x <- 1:10
q <- 0.6
beta <- 0.8
ddweibull(x, q, beta)
t <- qdweibull(0.99, q, beta)
t
pdweibull(t, q, beta)
#
x <- 0:10
ddweibull(x, q, beta, zero=TRUE)
t <- qdweibull(0.99, q, beta, zero=TRUE)
t
pdweibull(t, q, beta, zero=TRUE)

# Ex.2
q <- 0.4
beta <- 0.7
n <- 100
x <- rdweibull(n, q, beta)
tabulate(x)/sum(tabulate(x))
y <- 1:round(max(x))
# compare with
ddweibull(y, q, beta)

```

---

Discrete Weibull (Type 3)

*The type 3 discrete Weibull distribution*

---

**Description**

Probability mass function, distribution function, quantile function, random generation, and hazard function for the type 3 discrete Weibull distribution with parameters  $c$  and  $\beta$

**Usage**

```

ddweibull3(x, c, beta)
pdweibull3(x, c, beta)
qdweibull3(p, c, beta)
rdweibull3(n, c, beta)
hdweibull3(x, c, beta)

```

**Arguments**

x	vector of values/quantiles
p	vector of probabilities
c	first parameter

beta	second parameter
n	sample size

**Details**

The type 3 discrete Weibull distribution is characterized by the following cumulative distribution function:  $F(x; c, \beta) = 1 - \exp(-c \sum_{j=0}^{x+1} j^\beta)$ , for  $x = 0, 1, 2, \dots$ , with  $c > 0$  and  $\beta \geq -1$ .

**Value**

ddweibull3 gives the probability function, pdweibull3 gives the distribution function, qdweibull3 gives the quantile function, hdweibull3 gives the hazard function, and rdweibull generates random values.

**Author(s)**

Alessandro Barbiero

**Examples**

```
# ddweibull3
x <- 0:10
c <- 0.3
beta <- 0.75
p <- ddweibull3(x, c, beta)
p
plot(x, p, type="b", ylab=expression(P(X)==x))
# pdweibull3
x <- 0:10
c <- 0.5
beta <- 0.5
p <- pdweibull3(x, c, beta)
p
cumsum(ddweibull3(x, c, beta))
plot(x, p, type="s", ylab=expression(P(X<=x)))
# qdweibull3
p <- c(1:9)/10
p
c <- 0.1
beta <- 0.5
qdweibull3(p, c, beta)
pdweibull3(10, c, beta)
pdweibull3(9, c, beta)
# rdweibull3
n <- 20
c <- 0.25
beta <- -0.25
x <- rdweibull3(n, c, beta)
x
beta <- 0
x <- rdweibull3(n, c, beta)
x
```

```

beta <- 0.25
x <- rdweibull3(n, c, beta)
x
n <- 1000
x <- rdweibull3(n, c, beta)
obs <- c(sum(x==0), tabulate(x))
obs <- obs/sum(obs)
theo <- ddweibull3(min(x):max(x), c, beta)
barplot(rbind(obs, theo), beside=TRUE, names.arg=min(x):max(x),
ylab="relative frequency/probability", col=1:2)
legend(24, 0.1, c("observed", "theoretical"), pch=15, col=1:2)
#hdweibull3
x <- 0:15
c <- 0.5
hn<-hdweibull3(x, c, beta = -0.5)
h0<-hdweibull3(x, c, beta = 0)
hp<-hdweibull3(x, c, beta = 0.5)
plot(x, hn, type="b", ylim = c(0, 1), ylab="hazard rate")
points(x, h0, type = "b", col=2)
points(x, hp, type = "b", col=3)
legend(11, 0.5, c("beta<0", "beta=0", "beta>0"), col=1:3, pch=21)

```

---

Edweibull

*Expected values*


---

### Description

First and second order moments, variance and expected value of the reciprocal for the type 1 discrete Weibull distribution

### Usage

```

Edweibull(q, beta, eps = 1e-04, nmax = 1000, zero = FALSE)
E2dweibull(q, beta, eps = 1e-04, nmax = 1000, zero = FALSE)
Vdweibull(q, beta, eps = 1e-04, nmax = 1000, zero = FALSE)
ERdweibull(q, beta, eps = 1e-04, nmax = 1000)

```

### Arguments

q	first parameter
beta	second parameter
eps	error threshold for the numerical computation of the expected value
nmax	maximum value considered for the numerical approximate computation of the expected value;
zero	TRUE, if the support contains 0; FALSE otherwise

**Details**

The expected value is numerically computed considering a truncated support: integer values smaller than or equal to  $2F^{-1}(1 - \text{eps}; q, \beta)$  are considered, where  $F^{-1}$  is the inverse of the cumulative distribution function (implemented by the function `qdweibull`). However, if such value is greater than `nmax`, the expected value is computed recalling the formula of the expected value of the corresponding continuous Weibull distribution (see the reference), adding 0.5. Similar arguments apply to the other moments.

**Value**

the (approximate) expected values of the discrete Weibull distribution: `Edweibull` gives the first order moment, `E2dweibull` the second order moment, `Vdweibull` the variance, `ERdweibull` the expected value of the reciprocal (only if zero is FALSE)

**Author(s)**

Alessandro Barbiero

**References**

M. S. A. Khan, A. Khaliq, and A. M. Abouammoh (1989) On estimating parameters in a discrete Weibull distribution, *IEEE Transactions on Reliability*, 38(3), pp. 348-350

**Examples**

```
q <- 0.75
beta <- 1.25
Edweibull(q, beta)
E2dweibull(q, beta)
Vdweibull(q, beta)
ERdweibull(q, beta)
# if beta=0.75...
beta <- 0.75
Edweibull(q, beta)
Edweibull(q, beta, nmax=100)
# here above, the approximation through the continuous model intervenes
# if beta=1...
beta <- 1
Edweibull(q, beta)
# which equals...
1/(1-q)
```

---

Edweibull3

*Expected values*


---

**Description**

First and second order moments for the type 3 discrete Weibull distribution

**Usage**

```
Edweibull3(c, beta, eps = 1e-04)
E2dweibull3(c, beta, eps = 1e-04)
```

**Arguments**

c	first parameter
beta	second parameter
eps	error threshold for the numerical computation of the expected value

**Details**

The expected values are numerically computed considering a truncated support: integer values smaller than or equal to  $2F^{-1}(1 - eps; c, \beta)$ , where  $F^{-1}$  is the inverse of the cumulative distribution function (implemented by the function [qdweibull3](#))

**Value**

the (approximate) expected values of the discrete Weibull distribution: Edweibull3 gives the first order moment, E2dweibull3 the second order moment

**Author(s)**

Alessandro Barbiero

**Examples**

```
c <- 0.4
beta <- 0.25
Edweibull3(c,beta)
c <- 0.4
beta <- -0.75
Edweibull3(c, beta) # may require too much time
Edweibull3(c, beta, eps=0.001) # try with a smaller eps->worse approximation
c <- rep(0.1, 11)
beta <- (0:10)/10
Edweibull3(c, beta)
c <- rep(0.5, 11)
beta <- (-5:5)/10
Edweibull3(c,beta)
# E2dweibull3
c <- 0.4
beta <- 0.25
E2dweibull3(c, beta)
c <- rep(0.1, 11)
beta <- (0:10)/10
Edweibull3(c, beta)
c <- rep(0.8, 11)
beta <- (-5:5)/11
E2dweibull3(c, beta)
```



---

estdweibull                      *Estimation of parameters*

---

**Description**

Estimation of the parameters of the type 1 discrete Weibull distribution

**Usage**

```
estdweibull(x, method = "ML", zero = FALSE, eps = 1e-04, nmax=1000)
```

**Arguments**

x	the vector of sample values
method	"ML" for the maximum likelihood method; "M" for the method of moments; "P" for the method of proportions
zero	TRUE, if the support contains 0; FALSE otherwise
eps	error threshold for the computation of the moments of the distribution
nmax	maximum value considered for the numerical computation of the expected value

**Value**

the vector of the estimates of  $q$  and  $\beta$

**Author(s)**

Alessandro Barbiero

**See Also**

[ddweibull](#)

**Examples**

```
# Ex1
n <- 10
q <- 0.5
beta <- 0.8
x <- rdweibull(n, q, beta)
estdweibull(x, "ML") # maximum likelihood method
# it may return some harmless warnings
# that depend on the optimization function used in the maximization routine
estdweibull(x, "M") # method of moments
estdweibull(x, "P") # method of proportion
# the estimates provided by the three methods may be quite different
# from the true values... and to each other
# change the sample size
n <- 50
```

```

q <- 0.5
beta <- 0.8
x <- rdweibull(n, q, beta)
estdweibull(x, "ML") # maximum likelihood method
estdweibull(x, "M") # method of moments
estdweibull(x, "P") # method of proportion
# the estimates should be (on average) closer to the true values
# ...and to each other

# When the estimation methods fail...
# Ex2
# only 1s and 2s
x <- c(1,1,1,1,1,1,1,2,2,2,2)
estdweibull(x, "ML") # fails!
estdweibull(x, "M") # fails!
estdweibull(x, "P") # fails!

# Ex3
# no 1s
x <- c(2,2,3,4,5,5,5,6,6,8,10)
estdweibull(x, "ML") # works
estdweibull(x, "M") # works
estdweibull(x, "P") # fails!

```

---

estdweibull3

*Estimation of parameters*


---

### Description

Estimation of the parameters of the type 3 discrete Weibull distribution

### Usage

```
estdweibull3(x, method = "P", eps = 1e-04)
```

### Arguments

x	the vector of sample values
method	"ML" for the maximum likelihood method; "M" for the method of moments; "P" for the method of proportions
eps	error threshold for the computation of the moments of the distribution

### Value

the vector of the estimates of  $c$  and  $\beta$

### Author(s)

Alessandro Barbiero

**See Also**[ddweibull3](#)**Examples**

```

# Ex1
x <- c(0,0,0,0,0,1,1,1,1,1,1,1,2,2,2,2,3,3,4,6)
estdweibull3(x, "P")
estdweibull3(x, "ML")
estdweibull3(x, "M")
# Ex 2
n <- 20
c <- 1/3
beta <- 2/3
x <- rdweibull3(n, c, beta)
estdweibull3(x, "P")
par <- estdweibull3(x, "ML")
par
-loglikedw3(par, x)
par <- estdweibull3(x, "M")
par
lossdw3(par, x)
n <- 50
x <- rdweibull3(n, c, beta)
estdweibull3(x, "P")
estdweibull3(x, "ML")
estdweibull3(x, "M")
n <- 100
x <- rdweibull3(n, c, beta)
estdweibull3(x, "P")
estdweibull3(x, "ML")
estdweibull3(x, "M")
# Ex 3: a piece of simulation study
nSim <- 50
n <- 50
c <- 0.2
beta <- 0.7
par <- matrix(0, nSim, 2)
for(i in 1:nSim)
{
  x <- rdweibull3(n, c, beta)
  par[i,] <- estdweibull3(x, "ML")
}
op <- par(mfrow = c(1,2))
boxplot(par[,1], xlab=expression(hat(c)[ML]))
abline(h = c)
boxplot(par[,2], xlab=expression(hat(beta)[ML]))
abline(h = beta)
op <- par()

```

---

loglikedw	<i>Loglikelihood function</i>
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---

**Description**

Loglikelihood function (changed in sign) for the type 1 discrete Weibull distribution

**Usage**

```
loglikedw(par, x, zero = FALSE)
```

**Arguments**

par	the vector of parameters, $q$ and $\beta$
x	the vector of sample values
zero	TRUE, if the support contains 0; FALSE otherwise

**Value**

the value of the loglikelihood function (changed in sign) for the observed sample  $x$  under the parameters  $par$

**Author(s)**

Alessandro Barbiero

**See Also**

[estdweibull](#)

**Examples**

```
x <- c(1,1,1,2,2,2,2,2,3,4,4,5,6,8)
-loglikedw(c(0.8, 1), x) # loglikelihood function for q=0.8 and beta=1
-loglikedw(c(0.4, 2), x) # loglikelihood function for q=0.4 and beta=2
par <- estdweibull(x, "ML")# parameter estimates derived by the ML method
par
-loglikedw(par, x) # the maximum value of the loglikelihood function
```

---

loglikedw3	<i>Loglikelihood function</i>
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---

**Description**

Loglikelihood function (changed in sign) for the type 3 discrete Weibull distribution

**Usage**

```
loglikedw3(par, x)
```

**Arguments**

par	the vector of parameters, $c$ and $\beta$
x	the vector of sample values

**Value**

the value of the loglikelihood function (changed in sign) for the observed sample  $x$  under the parameters  $par$

**Author(s)**

Alessandro Barbiero

**See Also**

[estdweibull3](#)

**Examples**

```
n <- 20
c <- 1/3
beta <- 2/3
x <- rdweibull3(n, c, beta)
par <- estdweibull3(x, "ML")
par
-loglikedw3(par, x)
```

---

 Iosssdw

*Loss function*


---

**Description**

Loss function for the method of moments (type 1 discrete Weibull)

**Usage**

```
Iosssdw(par, x, zero = FALSE, eps = 1e-04, nmax=1000)
```

**Arguments**

par	vector of parameters $q$ and $\beta$
x	the vector of sample values
zero	TRUE, if the support contains 0; FALSE otherwise
eps	error threshold for the numerical computation of the expected value
nmax	maximum value considered for the numerical computation of the expected value

**Details**

The loss function is given by  $L(x; q, \beta) = [m_1 - E(X; q, \beta)]^2 + [m_2 - E(X^2; q, \beta)]^2$ , where  $E(\cdot)$  denotes the expected value,  $m_1$  and  $m_2$  are the first and second order sample moments respectively.

**Value**

the value of the quadratic loss function

**Author(s)**

Alessandro Barbiero

**See Also**

[Edweibull](#)

**Examples**

```
x <- c(1,1,1,1,1,2,2,2,3,4)
Iosssdw(c(0.5, 1), x)
par <- estdweibull(x, "M") # parameter estimates derived by the method of moments
par
Iosssdw(par, x) # the loss is zero using these estimates
```

---

lossdw3	<i>Loss function</i>
---------	----------------------

---

**Description**

Loss function for the method of moments (type 3 discrete Weibull)

**Usage**

```
lossdw3(par, x, eps = 1e-04)
```

**Arguments**

par	vector of parameters $q$ and $\beta$
x	the vector of sample values
eps	error threshold for the numerical computation of the expected value

**Details**

The loss function is given by  $L(x; c, \beta) = [m_1 - E(X; c, \beta)]^2 + [m_2 - E(X^2; c, \beta)]^2$ , where  $E(\cdot)$  denotes the expected value,  $m_1$  and  $m_2$  are the first and second order sample moments respectively.

**Value**

the value of the quadratic loss function

**Author(s)**

Alessandro Barbiero

**See Also**

[Edweibull3](#)

**Examples**

```
n <- 25
c <- 1/3
beta <- 2/3
x <- rdweibull3(n, c, beta)
par <- estdweibull3(x, "M")
par
lossdw3(par, x)
```

---

varFisher	<i>Observed Fisher information matrix</i>
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---

**Description**

Observed Fisher information matrix on a sample from the type 1 discrete Weibull distribution

**Usage**

```
varFisher(x, zero = FALSE)
```

**Arguments**

x	a vector of sample values
zero	TRUE, if the support contains 0; FALSE otherwise

**Value**

a list of two matrices: the observed Fisher information matrix, and its inverse, which contains asymptotic variances and covariances of the maximum likelihood estimators of  $q$  and  $\beta$

**Author(s)**

Alessandro Barbiero

**See Also**

[estdweibull](#)

**Examples**

```
x <- rdweibull(100, 2/3, 3/2)
estdweibull(x, "ML")
IF <- varFisher(x)[[2]]
diag(IF)
diag(IF)/length(x)
# asymptotic variances of the ML estimators directly estimated from the sample
```



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